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BENDING STRENGTH OF COLD-FORMED STEEL LONGITUDINALLY REINFORCED BEAM WEBS

By Phung Nguyen, ¹A.M. ASCE, and Wei-Wen Yu, ²F. ASCE

INTRODUCTION

Thin-walled, cold-formed steel beams having relatively large depths are increasingly used in building construction and other structural systems (11,13). When such deep beams are subjected to bending stress, shear stress, bearing stress, and combinations thereof, the buckling of web elements becomes more important. For the case of beam members subjected to bending, the buckling of the web element may occur prior to flange yielding. In order to improve the structural efficiency of the thin web, longitudinal stiffeners may be used. Unfortunately, little work has been done to study the structural behavior of cold-formed steel beam webs with longitudinal stiffeners even though the strength of longitudinally stiffened plate girders has been investigated previously (1,2). For this reason, an investigation of cold-formed steel beam webs having longitudinal stiffeners has been conducted at the University of Missouri-Rolla (UMR) as a part of the overall web study under the sponsorship of the American Iron and Steel Institute (AISI) (8).

In this paper the experimental results of 64 beam specimens are discussed along with the development of empirical formulas to be used for evaluating the required minimum rigidity of longitudinal stiffeners and the ultimate bending capacity of cold-formed steel beam members.

LITERATURE SURVEY

The buckling problem of a longitudinally stiffened plate subjected to pure bending has been studied by many investigators (4,5,9). The critical elastic buckling stress can be computed by the following equation:

$$f_{cr} = k\pi^2 E / [12(1-\mu^2) (h/t)^2] \quad (1)$$

in which k is the buckling coefficient, E is the modulus of elasticity, μ is the Poisson's ratio, h is the depth of web element, and t is the thickness of plate.

In general, the buckling coefficient is a function of the aspect ratio of the plate element, the edge support conditions, the location of the longitudinal stiffener, and the stiffener parameters as discussed in the subsequent paragraphs.

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In 1940, Massonet (5) found that for a plate having a longitudinal stiffener at the quarter depth of the web, the buckling coefficient, k , depends on the aspect ratio and the stiffener parameters which are expressed by the ratios $\gamma = EI_s / Dh$ and $\delta = A_s / ht$, in which D is the flexural rigidity of the plate, I_s is the moment of inertia of the longitudinal stiffener with respect to its centroidal axis parallel to the plane of the web, and A_s is the cross sectional area of the longitudinal stiffener.

The buckling problem of a longitudinally stiffened plate under pure bending was further studied by Dubas (4) and Rockey (9). They indicated that the relationship between the buckling coefficient, k , and the stiffener parameter γ and δ is shown in Fig. 1 for a stiffened plate with simple supports. This figure reveals that the value of k increases with the stiffener rigidity until the stiffener has sufficient rigidity to form a nodal line. Thereafter, a further increase in the moment of inertia of a stiffener will not lead to a substantial increase in the value of the buckling coefficient. The maximum value of k will be termed as the limiting value, k_0 , and the value of γ which a longitudinal stiffener must possess in order to provide this value of k_0 will be defined as γ_0 .

Figure 2 is a plot of the limiting buckling coefficient, k_0 , versus the location of a stiffener for the simply supported stiffened plates. the best location of the longitudinal stiffener was found to be $0.2h$ measured from the compressive edge. For this location, the maximum value of buckling coefficient is equal to 129.4.

Based on the bending behavior of a plate having a longitudinal stiffener located at $0.2h$ from the compressive edge, Massonet (5) proposed the following formula for the required minimum stiffener γ_0 :

$$\gamma_0 = 3.87 + 5.1\alpha + (8.82 + 77.6\delta)\alpha^2 \quad (2)$$

The required minimum stiffener rigidity given by the above equation is difficult to use for the design purpose because the designers must first arrive at a stiffener geometry according to the required value of γ_0 and the area ratio $\delta = A_s / ht$. In addition, the design of longitudinal stiffeners also depends on the aspect ratio of the plate element. Therefore, Eq. 2 is not suitable for the design of cold-formed steel structures because a constant cross-sectional shape of the longitudinal stiffener is often used for members of different lengths. To overcome this disadvantage, Rockey (2) proposed another expression, in 1963, for the required minimum rigidity which is independent of the aspect ratio. This expression is given by the following formula:

$$\gamma_0 = 43.4 + 381\delta + 1080\delta^2 \quad (3)$$

In order to determine the required minimum rigidity of the longitudinal stiffener from the above expression, the geometry of the stiffener and the location of its neutral axis for bending must be assumed. Moreover, Eq. 3 was developed on the basis of the buckling analysis of a plate element. It can not be used for a plate in the postbuckling range.

Recently, the strength of thin-walled compression elements with longitudinal stiffeners have been studied at Correll University (3). It was found that the required moment of inertia of an intermediate stiffener for multiple-stiffened compression elements can be expressed as a function of $(w/t) \sqrt{F_y}$, in which w is the overall flat width between webs or between a web and an edge stiffener, t is the thickness of a plate element, and F_y is the yield point of the material. Consequently Eq. 4 was developed to compute the required minimum stiffener rigidity in the postbuckling range.

$$I_s / t^4 = 0.581 (w/t) \sqrt{F_y} - 285 \quad (4)$$

The ultimate bending capacities of hot-rolled shapes and plate girders have been studied extensively at Lehigh University (1). In 1963, a research project was initiated at this institution with the general objective of determining the contribution of longitudinal stiffeners to the static load carrying capacity of plate girders (1,2). Based on his analytical study, Cooper concluded that the use of a longitudinal stiffener in the web can increase the maximum allowable slenderness ratio of the web from 170 to 400 for mild steel (1). Tests have shown that an adequately proportioned longitudinal stiffener located at $0.2h$ from the compression flange can prevent the loss in strength due to web buckling for webs having an overall slenderness ratio up to 450 (2). In this case, ordinary linear beam theory can be used to predict the maximum stress in the compression flange.

Plate girders with larger slenderness ratios are likely to require two or more longitudinal stiffeners to eliminate the loss of strength caused by web buckling. In general, the increase in bending strength of a girder with a longitudinally stiffened web is limited because the contribution of the web to the overall bending strength of the entire section is small. Based on his research findings, Cooper has provided a method for predicting the increase in bending strength of a plate girder with a longitudinal stiffener (1,2). In addition, longitudinal stiffeners have been found to be useful in improving the fatigue resistance of plate girders. It was pointed out by Yen and Mueller (12) that the improvement of the fatigue strength was primarily due to the reduction of the lateral deformation of the web under cyclic loading. This reduced lateral deformation minimizes the fatigue cracking at the web-to-flange junction.

Because the research work described above was on plate girders with attached stiffeners, this study deals with an experimental investigation of cold-formed steel beams having longitudinal rolled-in stiffeners in their webs.

EXPERIMENTAL INVESTIGATION

The objective of the experimental investigation has been to determine experimentally the required minimum rigidity of longitudinal stiffeners and the ultimate bending capacity of beams having longitudinally reinforced webs.

In this investigation, 64 beam specimens having longitudinally reinforced webs were tested under a pure bending condition. The longitudinal stiffeners were formed into a V-shape at $1/5$ of the web depth measured from the compression flange. Figures 3 and 4 show the configuration of the longitudinal stiffeners used in channels and hat sections. The test

specimens consisted of 60 built-up members fabricated from channels (Fig. 3) and 4 hat sections (Fig. 4). The channels were connected by $3/4 \times 3/4 \times 1/8$ in. (19.05 x 19.05 x 3.23 mm) angles at the compression flange and by $1/8 \times 3/4$ in. (3.23 x 19.05 mm) rectangular bars at the tension flange. The intervals between braces were close enough to prevent lateral buckling of each individual channel. For the hat sections, the tension flanges were connected by $1/8 \times 3/4$ in. (2.23 x 19.05 mm) rectangular bars. Hat sections were used for the larger flat-width ratios of the compression flange. Transverse stiffeners made of small channel sections were provided at the locations of the applied concentrated loads and at the supports (Fig. 5 and 6). They were used to prevent the premature failure due to the excessive contact bearing stress on the web element. These stiffeners were connected to the beam webs by using $3/4$ inch (19.05 mm) diameter bolts.

The actual cross-sectional dimensions of the built-up members and hat sections are given in Refs. 7 and 8. Tables 1 and 2 tabulates the pertinent parameters of all test specimens. The beam specimens tested in this investigation possessed the following ranges of parameters:

1. h/t ratios of webs: 200.28 to 357.54
2. I_s/t^4 ratios of stiffeners: 0 to 1479.81
3. w/t ratios of compression flange: 30.22 to 316.64
4. yield points of steels: 43.66 to 51.24 ksi (301.25 to 353.56 MN/m²).

In the above expressions, h = clear distance between flanges measured along the plane of the web, t = thickness of the steel, I_s = moment of inertia of the longitudinal stiffener with respect to its own centroidal axis, and w = flat width of the compression flange.

Each beam specimen was tested as a simply supported beam under two concentrated loads as shown in Figs. 5 and 6. A detailed description of the test procedure is presented in Refs. 6 and 7. All the beams were tested to failure as shown in Figs. 7 and 8. The failure loads, $(P_u)_{test}$ as well as other experimental data are recorded in Tables 3 and 4.

EVALUATION OF TEST RESULTS

A. Required Minimum Rigidity of Longitudinal Stiffeners - The effect of the longitudinal stiffener on the ultimate moment capacity is represented by the ratio of $(M_u)_{test}/(M_o)_{test}$ which is given in Table 8. In this ratio, $(M_u)_{test}$ is the tested bending moment of a beam specimen having a longitudinally reinforced web, and $(M_o)_{test}$ is the tested bending moment of a beam with an unreinforced web having the same cross-sectional dimensions. Figure 9 is a plot of the ratios of $(M_u)_{test}/(M_o)_{test}$ versus the parameter I_s/t^4 of the longitudinal stiffener rigidity for beam specimens having h/t ratios equal to 200, 275 and 325. This figure indicates that the moment ratio increases as the value of I_s/t^4 increases up to a limiting ratio of I_{o0}/t^4 , beyond which a further increase in the value of I_s/t^4 will not affect the moment ratio of $(M_u)_{test}/(M_o)_{test}$. For this reason the minimum moment of inertia of the longitudinal stiffener can be determined by the ratio of I_{o0}/t^4 .

The value of I_{o0}/t^4 has been studied on the basis of the experimental data obtained from this investigation. It was found that the I_{o0}/t^4 ratio depends on the ratio c/t of the web and the yield point of steel, where c is the clear distance between the neutral axis and the compression flange.

The values of $\sqrt{I_o/t^4}$ are given in Figure 9 for the beam specimens having various h/t ratios. The values of I_o/t^4 and the ratios of c/t for three groups of beam specimens are listed in the following table.

REQUIRED MINIMUM MOMENT OF INERTIA
FOR LONGITUDINAL STIFFENERS

(c/t)	(c/t) $\sqrt{F_y}$	$\sqrt{I_o/t^4}$	I_o/t^4
101.70	727.99	12.48	155.82
137.34	983.11	17.07	291.38
157.80	1129.57	19.02	361.60

Based on the experimental data listed in the above table, Eq. 5 has been developed to determine the required minimum rigidity of longitudinal stiffeners to be used for cold-formed steel beam members when the stiffener is located 1/5 depth of the web measured from the compression flange. This equation can be used for a beam web having a h/t ratio from 200 to about 325.

$$I_o/t^4 = 0.515 (c/t) \sqrt{F_y} - 217 \quad (5)$$

Figure 10 is a plot of I_o/t^4 versus the parameter $(c/t) \sqrt{F_y}$, from which Eq. 5 has been derived.

B. Ultimate Bending Strength of Beam Webs - The ultimate bending capacities of cold-formed steel beams having longitudinally reinforced webs are determined by two different design approaches: Postbuckling strength method and reduction in moment resistance method.

(a) **Postbuckling Strength of Beam Webs** -- When cold-formed steel beams having longitudinally reinforced webs are subjected to bending, the post-buckling strength of the web can be represented by the ratio of the tested ultimate load, $(P_u)_{test}$, and the theoretical load due to web buckling, $(P_{cr})_{theo}$. The latter is computed from the moment capacity of the beam specimen on the basis of the critical buckling stress given by Eq. 6 and the sectional properties by using the effective area of the compression flange and the full areas of the web and the tension flange.

$$f_{cr} = k_o \pi^2 E / [12 (1-\mu^2) (h/t)^2] \quad (6)$$

In Eq. 6, k_o = buckling coefficient for pure bending of the reinforced web element with a longitudinal stiffener at 1/5 of the web depth measured from the compression flange = 129.4. The other symbols were previously defined. The values of $(P_u)_{test} / (P_{cr})_{theo}$ are listed in Table 5 for the beam specimens having adequately stiffened web (i.e., $I_s/I_o \geq 1.0$ in Table 2).

Based on the experimental data, it was found that the post-buckling strength of the longitudinally reinforced web elements is a function of three significant parameters. They are the h/t ratio of the web element, the w/t ratio of the compression flange, and the yield point of steel. From an in-depth study of each of the aforementioned parameters, it was found that the post-buckling strength increases as the h/t ratio and F_y increase. However, an increase in the

w/t ratio results in a reduction of the postbuckling strength. The variation of the postbuckling strength versus various parameters are shown graphically in Figs. 11, 12 and 13.

Consequently, the postbuckling strength of the adequately reinforced beam webs with longitudinal stiffeners can be determined by the following formula:

$$\Phi = \alpha_1 \alpha_2 \alpha_3 \geq 1.0 \quad (7)$$

in which

$$\begin{aligned} \Phi &= \text{postbuckling strength factor} \\ &= (P_u)_{\text{test}} / (P_{cr})_{\text{theo}} \end{aligned}$$

$$\alpha_1 = 6.39 (10^{-3}) (h/t) - 0.697 \quad (7a)$$

$$\begin{aligned} \alpha_2 &= 1.18 - 0.16 (w/t)/(w/t)_{\text{lim}}, \text{ when } (w/t)/(w/t)_{\text{lim}} \leq 2.0 \\ &= 0.86, \text{ when } (w/t)/(w/t)_{\text{lim}} \geq 2.0 \end{aligned} \quad (7b)$$

$$\alpha_3 = 0.50 (F_y/33) + 0.24 \quad (7c)$$

$$(w/t)_{\text{lim}} = 171 \sqrt{f} \text{ according to Section 2.3.1.1 of the AISI Specification (10).}$$

$$f = \text{actual stress in the compression flange computed on the basis of the effective design width, ksi.}$$

Equations 7a through 7c were developed from a regression analysis.

The comparison of the tested and computed postbuckling strength factors is represented graphically in Fig. 14, which indicates that Eq. 7 adequately predicts the postbuckling strength to within $\pm 10\%$ of the tested value.

The moment capacities of the beam specimens used in the test program, $(M_u)_{\text{comp}}$, were determined by the following two equations, whichever is smaller.

$$(M_u)_{cr} = \Phi S'_x f_{cr} \quad (8)$$

$$M_y = S_x F_y \quad (9)$$

In the above equations, $(M_u)_{cr}$ and M_y are the computed bending moments governed by the web element and the flange respectively, S_x is the section modulus determined by the yield point F_y , and S'_x is the section modulus based on the critical buckling stress f_{cr} computed by using Eq. 1.

The accuracy of this method is indicated by the ratio of $(M_u)_{\text{test}} / (M_u)_{\text{comp}}$ listed in Table 6. The moment ratios range from 0.987 to 1.072 with a mean value of 1.030 and a standard deviation of 0.018.

(b) Reduction in Moment Resistance -- Because of the use of thin material for cold-formed steel beams, the compression portion of the

beam web with a large h/t ratio may buckle prior to the failure of the compression flange. Consequently, the compressive stress which the web would have resisted is, therefore, shifted to the compression flange. This behavior results in the reduction of the flange capacity. This reduction is represented by the ratio of $(P_y)_{test} / (P_y)_{theo}$ given in Table 5. In this ratio, $(P_y)_{theo}$ is the computed load for the beam section by considering the yield point of steel as the maximum bending stress.

Based on the test results obtained from this experimental investigation, Eq. 10 was derived for computing a reduction factor λ to be used for determining the moment resistance of the cold-formed steel beams with longitudinally reinforced webs having $h/t > 163.29 \sqrt{k_o/F_y}$.

$$\lambda = \gamma_1 \gamma_2 \leq 1.0 \quad (10)$$

$$\gamma_1 = 1.295 - 2.33 (10^{-3}) (h/t) \sqrt{F_y/k_o} \quad (10a)$$

$$\begin{aligned} \gamma_2 &= 1.15 - 0.10 (w/t)/(w/t)_{lim}, \text{ when } (w/t)/(w/t)_{lim} < 1.8 \\ &= 0.97, \text{ when } (w/t)/(w/t)_{lim} \geq 1.8 \end{aligned} \quad (10b)$$

In the above equations, λ represents the ratio of $(P_y)_{test} / (P_y)_{theo}$, k_o = buckling coefficient = 129.4, $(w/t)_{lim}$ and F_y were defined previously.

Equations 10a and 10b were developed from Figs. 15 and 16, which are the plots of the tested values of λ versus the parameters $(h/t) \sqrt{F_y/k_o}$ and $(w/t)/(w/t)_{lim}$, respectively.

By using Eq. 10, the moment capacities of beam specimens having adequately stiffened webs, $(M_u)_{comp}$, were evaluated by using the following equation:

$$(M_u)_{comp} = \lambda M_y = \lambda S_x F_y \quad (11)$$

Table 7 gives the values of $(M_u)_{comp}$ and the ratios of $(M_u)_{comp} / (M_u)_{test}$. Good agreement was obtained for the moment ratios which vary from 0.971 to 1.022 with an average value of 0.998 and a standard deviation of 0.018.

SUMMARY

An experimental investigation was conducted to study the ultimate bending capacity of cold-formed steel beams having reinforced webs with longitudinal stiffeners. Based on results of 64 beam tests, empirical design formulas were developed to determine the required minimum rigidity of longitudinal stiffeners and to compute the ultimate bending moments of the beams having adequately reinforced webs. The design method for determining the bending capacity of beam webs involves the use of either a postbuckling strength factor or a reduction factor as discussed in the text.

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APPENDIX II. NOTATIONS

The following symbols are used in this paper:

- A_s = cross-sectional area of longitudinal stiffener, in inches²;
- E = modulus of elasticity, in kips per square inch;
- F_y = yield point, in kips per square inch;
- f = actual stress in compression flange, in kips per sq. inch;
- f_{cr} = critical buckling stress in web, in kips per sq. inch;
- h = clear distance between flanges measured along the plane of the web, in inches;
- I_o = minimum moment of inertia of a longitudinal stiffener with respect to its own centroidal axis parallel to the plane of the web, in inches⁴;
- I_s = moment of inertia of a longitudinal stiffener with respect to its own centroidal axis parallel to the plane of the web, in inches⁴;
- k = buckling coefficient for a web in bending;
- k_o = maximum value of buckling coefficient for a longitudinally reinforced web in bending;
- $(M_o)_{test}$ = tested ultimate bending moment for beam specimens having unreinforced webs, in inch-kips;
- $(M_u)_{comp}$ = computed ultimate bending moment, in inch-kips;
- $(M_u)_{cr}$ = computed bending moment based on Φ , in inch-kips;
- $(M_u)_{test}$ = tested ultimate bending moment, in inch-kips;
- M_y = computed bending moment governed by the flange, in inch-kips;
- $(P_{cr})_{test}^f$ = tested critical flange buckling load, in kips;
- $(P_u)_w$ = tested critical web buckling load, in kips;
- $(P_u)_{test}$ = failure load, in kips;
- $(P_y)_{test}$ = tested yield load, in kips;
- S_x = section modulus based on F_y , in inches³;
- S'_x = section modulus based on f_{cr} , in inches³;
- t = thickness of base metal, in inches;
- w = flat width, in inches;
- α_1 = postbuckling factor for h/t ;
- α_2 = postbuckling factor for $(w/t)/(w/t)_{lim}$;
- α_3 = postbuckling factor for F_y ;
- δ = ratio of stiffener-to-plate area;

APPENDIX II. NOTATIONS (continued)

- γ = ratio of stiffener-to-plate rigidity;
 γ_0 = required minimum value of the ratio of stiffener-to-plate rigidity;
 γ_1 = reduction factor for $(h/t)\sqrt{F_y/k_o}$;
 γ_2 = reduction factor for $(w/t)/(w/t)_{lim}$;
 λ = reduction factor for bending capacity;
 Φ = postbuckling strength factor, and
 μ = Poisson's ratio.

TABLE 1

PERTINENT PARAMETERS OF TEST SPECIMENS
FOR A STUDY OF LONGITUDINAL STIFFENERS

Beam Specimen No. (1)	h/t (2)	w/t (3)	(w/t) _{lim} (4)	F _y , in kips per square inch (5)	I _s /t ⁴ (6)	a, in inches (7)
B-1-1	214.01	31.66	31.37	51.24	0	26
B-1-2	210.10	32.48	31.26	51.24	0	26
B-1a-1	207.50	32.25	31.22	51.24	25.33	26
B-1a-2	206.14	31.02	31.28	51.24	24.82	26
B-1b-1	209.29	30.45	31.20	51.24	67.86	26
B-1b-2	205.73	30.79	31.22	51.24	65.56	26
B-1c-1	204.29	31.73	31.28	51.24	171.30	26
B-1c-2	204.46	31.59	31.30	51.24	171.30	26
B-1d-1	202.81	30.67	31.66	51.24	538.08	26
B-1b-2	202.96	32.45	31.91	51.24	692.22	26
B-2-1	279.64	30.65	31.43	51.24	0	35
B-2-2	281.60	31.42	31.33	51.24	0	35
B-2a-1	280.09	32.04	30.99	51.24	27.48	35
B-2a-2	275.97	31.74	31.11	51.24	24.16	35
B-2b-1	282.29	31.22	31.09	51.24	67.86	35
B-2b-2	275.45	30.40	31.07	51.24	63.79	35
B-2c-1	279.41	31.71	31.28	51.24	139.55	35
B-2c-2	278.12	31.13	31.27	51.24	136.67	35
B-2d-1	280.55	31.21	31.51	51.24	410.92	35
B-2d-2	277.90	30.58	31.54	51.24	399.52	35
B-2e-1	279.11	31.39	31.73	51.24	1097.44	35
B-2e-2	278.97	31.08	31.66	51.24	1048.56	35
B-3-1	317.19	30.57	31.35	51.24	0	40
B-3-2	321.04	31.25	31.33	51.24	0	40
B-3a-1	317.21	31.11	31.17	51.24	66.01	40
B-3a-2	319.16	32.02	31.20	51.24	65.56	40
B-3b-1	321.03	31.23	31.29	51.24	133.00	40
B-3b-2	325.77	32.24	31.26	51.24	139.63	40
B-3c-1	320.67	36.42	31.51	51.24	414.97	40
B-3c-2	320.67	31.31	31.42	51.24	439.52	40
B-3d-1	324.22	31.01	31.50	51.24	1236.77	40
B-3d-2	321.85	30.88	31.50	51.24	1015.95	40

Notes: 1. 1 in. = 25.4 mm

2. See Fig. 5 for definition of a.

3. I_s is the moment of inertia of longitudinal stiffener with respect to its centroidal axis parallel to the plane of the web.

TABLE 2
PERTINENT PARAMETERS OF BENDING TEST SPECIMENS

Beam Specimen No.	h/t	w/t	(w/t) _{lim}	F _y , in kips per square inch	I _s /I _o	a, in inches
(1)	(2)	(3)	(4)	(5)	(6)	(7)
B-1c-1	204.29	31.73	31.28	51.24	1.11	26
B-1c-2	204.46	31.59	31.30	51.24	1.11	26
B-1d-1	202.81	30.67	31.66	51.24	3.66	26
B-1d-2	202.96	32.45	31.91	51.24	4.80	26
B-2d-1	280.55	31.21	31.51	51.24	1.42	35
B-2d-2	277.90	30.58	31.54	51.24	1.41	35
B-2e-1	279.11	31.39	31.73	51.24	3.89	35
B-2e-2	278.97	31.08	31.66	51.24	3.70	35
B-3c-1	320.67	36.42	31.51	51.24	1.15	40
B-3c-2	320.67	31.31	31.42	51.24	1.21	40
B-3d-1	324.22	31.01	31.50	51.24	3.36	40
B-3d-2	321.85	30.88	31.50	51.24	2.79	40
B-4a-1	343.05	57.40	31.99	47.63	0.30	45
B-4a-2	331.80	55.79	31.99	47.63	0.31	45
B-4b-1	332.93	56.35	31.99	47.63	0.93	45
B-4b-2	330.07	55.88	31.99	47.63	0.91	45
B-4c-1	333.47	56.53	31.99	47.63	2.63	45
B-4c-2	332.83	56.04	31.99	47.63	2.36	45
B-5a-1	357.54	30.57	31.51	51.24	2.55	45
B-5a-2	357.27	31.06	31.57	51.24	2.51	45
B-6a-1	200.28	30.22	34.11	43.66	1.48	23
B-6a-2	200.94	31.01	34.28	43.66	1.40	23
B-7a-1	277.34	30.44	34.19	43.66	2.10	32
B-7a-2	273.92	29.42	34.04	43.66	2.10	32
B-8a-1	326.30	30.59	34.30	43.66	3.93	38
B-8a-2	326.42	31.86	34.32	43.66	4.57	38
B-9a-1	321.70	57.06	30.84	51.24	3.35	40
B-9a-2	321.27	58.02	30.84	51.24	3.39	40
H-1a-1	326.47	159.28	30.84	51.24	2.63	40
H-1a-2	324.16	158.00	30.84	51.24	2.68	40
H-2a-1	326.17	314.33	30.84	51.24	2.58	40
H-2a-2	328.17	316.64	30.84	51.24	2.89	40

Notes: 1. 1 in. = 25.4 mm.

2. See Fig. 5 for definition of a.

3. I_o is the required minimum moment of inertia of longitudinal stiffeners with respect to its centroidal axis.

4. I_s is the actual moment of inertia of longitudinal stiffeners with respect to its centroidal axis parallel to the plane of the web.

TABLE 3

EXPERIMENTAL DATA OF TEST SPECIMENS
FOR A STUDY OF LONGITUDINAL STIFFENERS

Beam Specimen	(P _{cr}) test, in kips	(P _y) test, in kips	(P _u) test, in kips	(M _u) test, in inch-kips	(M _o) test, in inch-kips	(M _u) test (M _o) test
No. (1)	(2)	(3)	(4)	(5)	(6)	(7)
B-1-1*	3.00	--	5.96	77.48	79.37	0.976
B-1-2*	2.50	--	6.25	81.25	79.37	1.024
B-1a-1	--	--	7.28	94.64	79.37	1.192
B-1a-2	--	--	7.17	93.21	79.37	1.174
B-1b-1	--	--	8.29	107.71	79.37	1.357
B-1b-2	--	--	8.04	104.52	79.37	1.317
B-1c-1	--	8.50	9.28	120.64	79.37	1.520
B-1c-2	--	8.65	9.03	117.39	79.37	1.479
B-1d-1	--	8.59	9.15	118.95	79.37	1.499
B-1d-2	--	8.85	9.25	120.25	79.37	1.515
B-2-1*	2.15	--	6.43	112.53	112.27	1.002
B-2-2*	2.50	--	6.40	112.00	112.27	0.998
B-2a-1	5.75	--	7.65	133.88	112.27	1.192
B-2a-2	6.00	--	7.89	138.08	112.27	1.230
B-2b-1	6.50	--	8.43	147.53	112.27	1.314
B-2b-2	7.00	--	8.51	148.93	112.27	1.327
B-2c-1	8.50	--	9.02	157.85	112.27	1.406
B-2c-2	8.75	--	8.92	156.10	112.27	1.390
B-2d-1	8.75	9.74	10.03	175.53	112.27	1.563
B-2d-2	8.50	9.80	10.39	181.83	112.27	1.620
B-2e-1	9.25	10.10	10.39	181.83	112.27	1.620
B-2e-2	9.00	10.00	10.36	181.30	112.27	1.615
B-3-1*	1.50	--	7.20	129.60	131.22	0.988
B-3-2*	1.75	--	7.38	132.84	131.22	1.012
B-3a-1	6.25	--	9.13	164.34	131.22	1.252
B-3a-2	5.75	--	8.58	154.44	131.22	1.177
B-3b-1	6.75	--	9.41	169.38	131.22	1.291
B-3b-2	6.75	--	9.90	178.20	131.22	1.358
B-3c-1	7.25	--	10.94	196.92	131.22	1.501
B-3c-2	7.75	--	11.31	203.58	131.22	1.551
B-3d-1	8.50	--	11.02	198.36	131.22	1.512
B-3d-2	8.50	--	11.14	200.52	131.22	1.528

* Beam specimens having flat webs without longitudinal stiffeners.

Note: 1 kip = 4.45kN; 1 in.-kip = 113 N-m

TABLE 4
EXPERIMENTAL DATA FOR BENDING TEST SPECIMENS

Beam	$(P_{cr})^w_{test}$	$(P_{cr})^f_{test}$	$(P_y)_{test}$	$(P_u)_{test}$	$(M_u)_{test}$
Specimen	in kips	in kips	in kips	in kips	in
No.					inch-kips
(1)	(2)	(3)	(4)	(5)	(6)
B-1c-1	--	--	8.50	9.28	120.64
B-1c-2	--	--	8.65	9.03	117.39
B-1d-1	--	--	8.59	9.15	118.95
B-1d-2	--	--	8.85	9.25	120.25
B-2d-1	8.75	--	9.74	10.03	175.53
B-2d-2	8.50	--	9.80	10.39	181.83
B-2e-1	9.25	--	10.10	10.39	181.83
B-2e-2	9.00	--	10.00	10.36	181.30
B-3c-1	7.25	--	--	10.94	196.92
B-3c-2	7.75	--	--	11.31	203.58
B-3d-1	8.50	--	--	11.02	198.36
B-3d-2	8.50	--	--	11.14	200.52
B-4a-1	7.50	7.75	--	8.74	196.65
B-4a-2	7.00	7.25	--	8.09	182.03
B-4b-1	7.75	8.00	--	8.75	196.88
B-4b-2	6.55	7.75	--	9.02	202.98
B-4c-1	7.00	8.25	--	9.90	222.75
B-4c-2	6.25	7.75	--	9.74	219.15
B-5a-1	8.50	--	--	14.38	230.08
B-5a-2	11.25	--	--	14.21	227.36
B-6a-1	--	--	5.95	6.30	69.00
B-6a-2	--	--	5.94	6.24	68.31
B-7a-1	6.75	--	7.00	7.47	119.52
B-7a-2	6.25	--	7.00	7.29	116.64
B-8a-1	6.00	--	--	7.22	137.18
B-8a-2	5.50	--	--	7.15	135.85
B-9a-1	7.75	10.75	--	12.35	222.30
B-9a-2	8.04	10.25	--	12.12	218.16
H-1a-1	7.02	1.50	--	9.60	153.60
H-1a-2	6.50	1.60	--	9.82	157.12
H-2a-1	6.75	.55	--	9.80	156.80
H-2a-2	7.50	.50	--	9.64	154.24

Note: 1 kip = 4.45 kN; 1 in. - kip = 113 N-m.

TABLE 5
COMPARISON OF EXPERIMENTAL AND THEORETICAL DATA FOR BENDING
TEST SPECIMENS HAVING ADEQUATELY STIFFENED WEBS

Beam Specimen No.	(1)	(2) $(P_{cr})^w$ in kips	(3) $(P_{cr})^f$ in kips	(4) $(P_y)_{theo}$, in kips	(5)		(6)		(7)		(8)		(9)	
					$(P_{cr})^w$ $(P_{cr})^w$	$(P_{cr})^w$ $(P_{cr})^w$	$(P_{cr})^f$ $(P_{cr})^f$	$(P_{cr})^f$ $(P_{cr})^f$	$(P_y)_{test}$ $(P_y)_{theo}$	$(P_y)_{test}$ $(P_y)_{theo}$	$(P_u)_{test}$ $(P_{cr})^w$	$(P_u)_{test}$ $(P_y)_{theo}$	$(P_u)_{test}$ $(P_y)_{theo}$	$(P_u)_{test}$ $(P_y)_{theo}$
B-1c-1		--	--	9.58	--	--	--	--	0.887	0.887	--	--	0.969	0.969
B-1c-2		--	--	9.58	--	--	--	--	0.903	0.903	--	--	0.943	0.943
B-1d-1		--	--	9.54	--	--	--	--	0.900	0.900	--	--	0.959	0.959
B-1d-2		--	--	9.54	--	--	--	--	0.928	0.928	--	--	0.970	0.970
B-2d-1		8.46	--	10.86	1.034	1.034	--	--	0.897	0.897	1.186	1.186	0.924	0.924
B-2d-2		8.66	--	10.92	0.982	0.982	--	--	0.897	0.897	1.200	1.200	0.951	0.951
B-2e-1		8.59	--	10.92	1.077	1.077	--	--	0.925	0.925	1.210	1.210	0.951	0.951
B-2e-2		8.62	--	10.95	1.044	1.044	--	--	0.913	0.913	1.202	1.202	0.946	0.946
B-3c-1		7.90	--	13.21	1.040	1.040	--	--	--	--	1.385	1.385	0.828	0.828
B-3c-2		7.96	--	13.35	0.974	0.974	--	--	--	--	1.421	1.421	0.847	0.847
B-3d-1		7.67	--	31.15	1.108	1.108	--	--	--	--	1.437	1.437	0.838	0.838
B-3d-2		7.83	--	31.24	1.086	1.086	--	--	--	--	1.423	1.423	0.841	0.841
B-4c-1		7.54	8.06	12.11	0.928	0.928	1.024	1.024	--	--	1.313	1.313	0.818	0.818
B-4c-2		7.77	8.44	12.47	0.804	0.804	0.918	0.918	--	--	1.254	1.254	0.781	0.781
B-5a-1		8.29	--	17.29	1.026	1.026	--	--	--	--	1.735	1.735	0.832	0.832
B-5a-2		8.27	--	17.23	1.360	1.360	--	--	--	--	1.718	1.718	0.825	0.825
B-6a-1		--	--	6.53	--	--	--	--	0.911	0.911	--	--	0.965	0.965
B-6a-2		--	--	6.57	--	--	--	--	0.904	0.904	--	--	0.950	0.950
B-7a-1		7.14	--	7.64	0.945	0.945	--	--	0.916	0.916	1.046	1.046	0.978	0.978
B-7a-2		7.19	--	7.50	0.869	0.869	--	--	0.933	0.933	1.014	1.014	0.972	0.972
B-8a-1		5.45	--	8.07	1.101	1.101	--	--	--	--	1.325	1.325	0.895	0.895
B-8a-2		5.41	--	8.02	1.017	1.017	--	--	--	--	1.316	1.316	0.892	0.892
B-9a-1		9.57	9.46	15.37	0.810	0.810	1.136	1.136	--	--	1.290	1.290	0.804	0.804
B-9a-2		9.62	9.13	15.32	0.836	0.836	1.123	1.123	--	--	1.260	1.260	0.791	0.791

TABLE 5
COMPARISON OF EXPERIMENTAL AND THEORETICAL DATA FOR BENDING
TEST SPECIMENS HAVING ADEQUATELY STIFFENED WEBS
(Continued)

Beam Specimen No.	(P _{cr}) ^w theo in kips	(P _{cr}) ^f theo in kips	(P _y) ^{theo} in kips	(P _{cr}) ^w test (P _{cr}) ^w theo	(P _{cr}) ^f test (P _{cr}) ^f theo	(P _y) ^{test} (P _y) ^{theo}	(P _u) ^{test} (P _{cr}) ^w theo	(P _u) ^{test} (P _y) ^{theo}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
H-1a-1	8.01	1.14	12.27	0.876	1.316	--	1.198	0.782
H-1a-2	8.15	1.14	12.23	0.798	1.379	--	1.205	0.803
H-2a-1	7.99	0.32	12.17	0.845	1.719	--	1.227	0.805
H-2a-2	7.93	0.32	12.17	0.946	1.563	--	1.216	0.784
Mean				0.978		0.910		
Standard Deviation				0.130		0.015		

Note: 1 kip = 4.5 kN; 1 in.-kip = 113 N-m.

TABLE 6
COMPARISON OF TESTED AND COMPUTED ULTIMATE MOMENT CAPACITIES
FOR BENDING TEST SPECIMENS BASED ON THE POSTBUCKLING STRENGTH METHOD

Beam Specimen No.	f_{cr} , in kips per square inch	S'_x , in cubic inches	$M_{cr} = S'_x f_{cr}$ in inch-kips	$(M_u)_{cr} = \phi M_{cr}$ in inch-kips	S_x , in cubic inches	F_y , in kips per square inch	$M_y = S_x F_y$, in inch-kips	$(M_u)_{test}$ in inch-kips	$\frac{(M_u)_{test}}{(M_u)_{comp}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
B-2d-1	38.89	3.71	147.99	168.41	3.71	51.24	190.10	175.53	1.042
B-2d-2	40.65	3.73	151.62	170.42	3.73	51.24	191.13	181.83	1.067
B-2e-1	40.30	3.73	150.32	169.56	3.73	51.24	191.13	181.83	1.072
B-2e-2	40.34	3.74	150.87	170.33	3.74	51.24	191.64	181.30	1.064
B-3c-1	30.53	4.66	142.27	194.63	4.64	51.24	237.75	196.92	1.012
B-3c-2	30.53	4.69	143.19	200.75	4.69	51.24	240.32	203.58	1.014
B-3d-1	29.87	4.62	138.00	197.20	4.62	51.24	236.73	198.36	1.006
B-3d-2	30.31	4.65	140.94	199.29	4.65	51.24	238.27	200.52	1.006
B-4c-1	28.23	6.01	169.66	209.87	5.72	47.63	272.44	222.75	1.061
B-4c-2	28.34	6.17	174.86	216.30	5.89	47.63	280.54	219.15	1.013
B-5a-1	24.56	5.40	132.62	219.35	5.40	51.24	276.70	230.08	1.049
B-5a-2	24.60	5.38	132.35	218.11	5.38	51.24	275.67	227.36	1.042
B-7a-1	40.82	2.80	114.30	114.99	2.80	43.66	122.25	119.52	1.039
B-7a-2	41.84	2.75	115.06	113.79	2.75	43.66	120.07	116.64	1.025
B-8a-1	29.49	3.51	103.51	134.36	3.51	43.66	153.25	137.18	1.021
B-8a-2	29.47	3.49	102.85	132.78	3.49	43.66	152.37	135.85	1.023
B-9a-1	30.34	5.68	172.33	210.41	5.40	51.24	276.70	222.30	1.057
B-9a-2	30.42	5.69	173.09	209.61	5.38	51.24	275.67	218.16	1.041
H-1a-1	32.37	3.96	128.19	155.62	3.83	51.24	196.25	153.60	0.987
H-1a-2	32.83	3.97	130.34	156.54	3.82	51.24	195.74	157.12	1.004
H-2a-1	32.43	3.94	127.77	154.86	3.80	51.24	194.71	156.80	1.013
H-2a-2	32.04	3.96	126.88	155.30	3.80	51.24	194.71	154.24	0.993

*The value of $(M_u)_{comp}$ is determined by $(M_u)_{cr}$ or M_y whichever is smaller.

- Notes: 1. 1 ksi = 6.9 MN/m²; 1 cu in. = 16.4 cm³; 1 in. - kip = 113 N-m
2. The values of S_x and S'_x were computed on the basis of F_y and f_{cr} , respectively.

TABLE 7

COMPARISON OF TESTED AND COMPUTED ULTIMATE MOMENT CAPACITIES
FOR BENDING TEST SPECIMENS BASED ON THE REDUCED MOMENT RESISTANCE METHOD

Beam Specimen No. (1)	S_x , in cubic inches (2)	F_y , in kips per square inch (3)	$M_y = S_x F_y$, in inch-kips (4)	λ (5)	$M'_y = \lambda M_y$, in inch-kips (6)	$(M_u)_{test}$, in inch-kips (7)	$\frac{(M_u)_{test}}{(M_u)_{comp}}^*$ (8)
B-2d-1	3.71	51.24	190.10	0.929	176.60	175.53	0.994
B-2d-2	3.73	51.24	191.13	0.935	178.71	181.83	1.017
B-2e-1	3.73	51.24	191.13	0.931	177.94	181.83	1.022
B-2e-2	3.74	51.24	191.64	0.932	178.61	181.30	1.015
B-3c-1	4.64	51.24	237.75	0.853	202.80	196.92	0.971
B-3c-2	4.69	51.24	240.32	0.866	208.12	203.58	0.978
B-3d-1	4.62	51.24	236.73	0.862	204.06	198.36	0.972
B-3d-2	4.65	51.24	238.27	0.866	206.34	200.52	0.972
B-4c-1	5.72	47.63	272.44	0.802	218.50	222.75	1.019
B-4c-2	5.89	47.63	280.54	0.804	225.55	219.15	0.972
B-5a-1	5.40	51.24	276.70	0.812	224.68	230.08	1.024
B-5a-2	5.38	51.24	275.67	0.811	223.57	227.36	1.017
B-7a-1	2.80	43.66	122.25	0.976	119.32	119.52	1.002
B-7a-2	2.75	43.66	120.07	0.983	118.03	116.64	0.988
B-8a-1	3.51	43.66	153.25	0.905	138.69	137.18	0.989
B-8a-2	3.49	43.66	152.37	0.902	137.44	135.85	0.988
B-9a-1	5.40	51.24	276.70	0.799	221.08	222.30	1.006
B-9a-2	5.38	51.24	275.67	0.799	220.26	218.16	0.990
H-1a-1	3.83	51.24	196.25	0.792	155.43	153.60	0.988
H-1a-2	3.82	51.24	195.74	0.795	155.61	157.12	1.010
H-2a-1	3.80	51.24	194.71	0.793	154.41	156.80	1.015
H-2a-2	3.80	51.24	194.71	0.789	153.63	154.24	1.004
Mean							0.998
Standard Deviation							0.018

* The value of $(M_u)_{comp}$ equals to M'_y .

Note: 1 cu in. = 16.4 cm³; 1 ksi = 6.9 MN/m²; 1 in.-kip = 113 N-m.

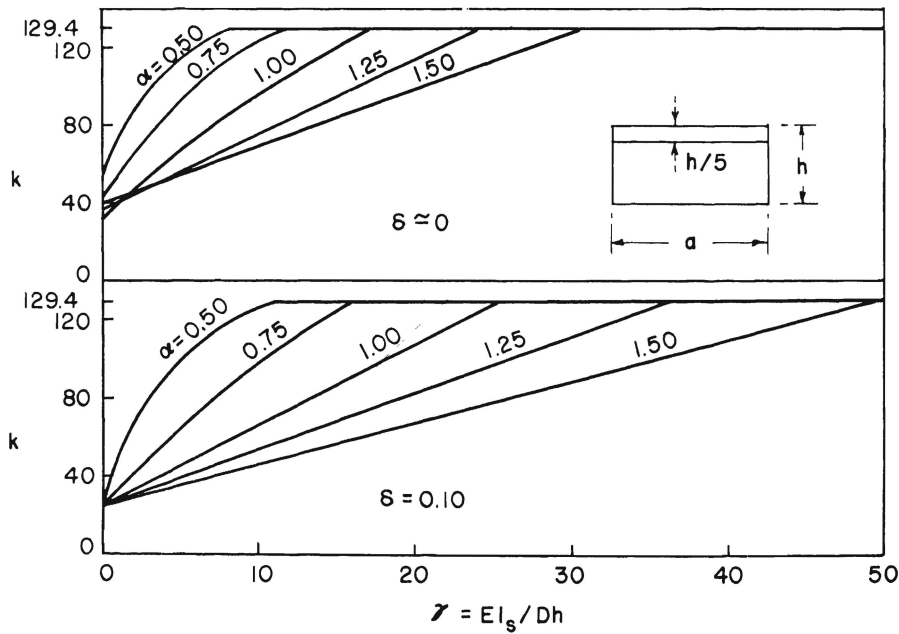


Fig.1 Variation of Buckling Coefficient versus Stiffener Rigidity for Simply Supported Longitudinally Stiffened Plates.

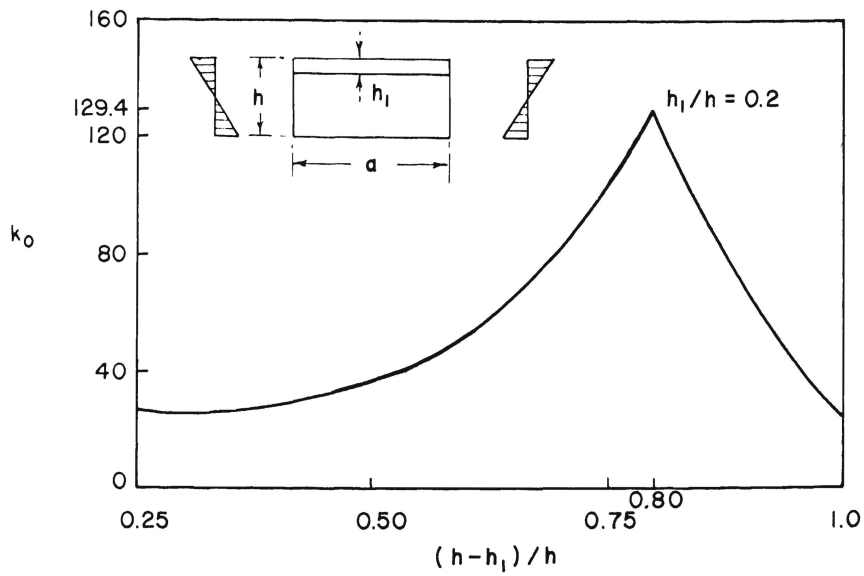


Fig.2 Variation of Maximum Buckling Coefficient versus the Location of Longitudinal Stiffeners.

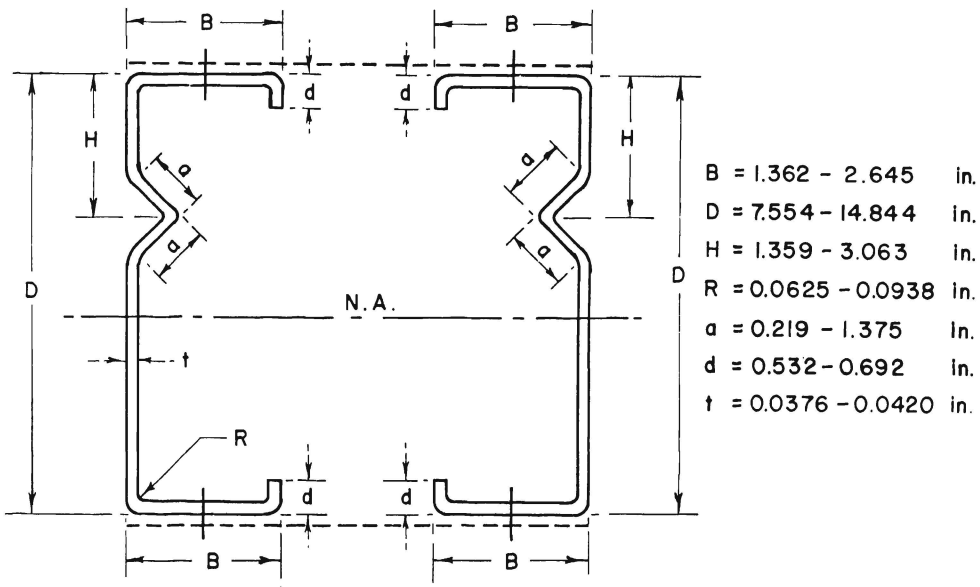


Fig. 3 Dimension of Channel Specimens

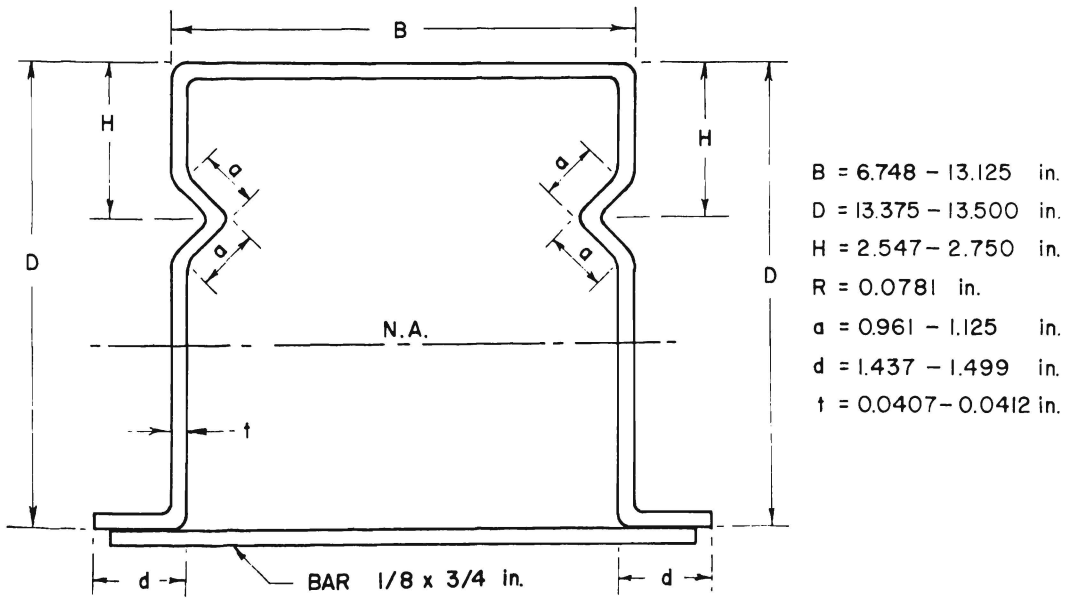


Fig. 4 Dimension of Hat Specimens

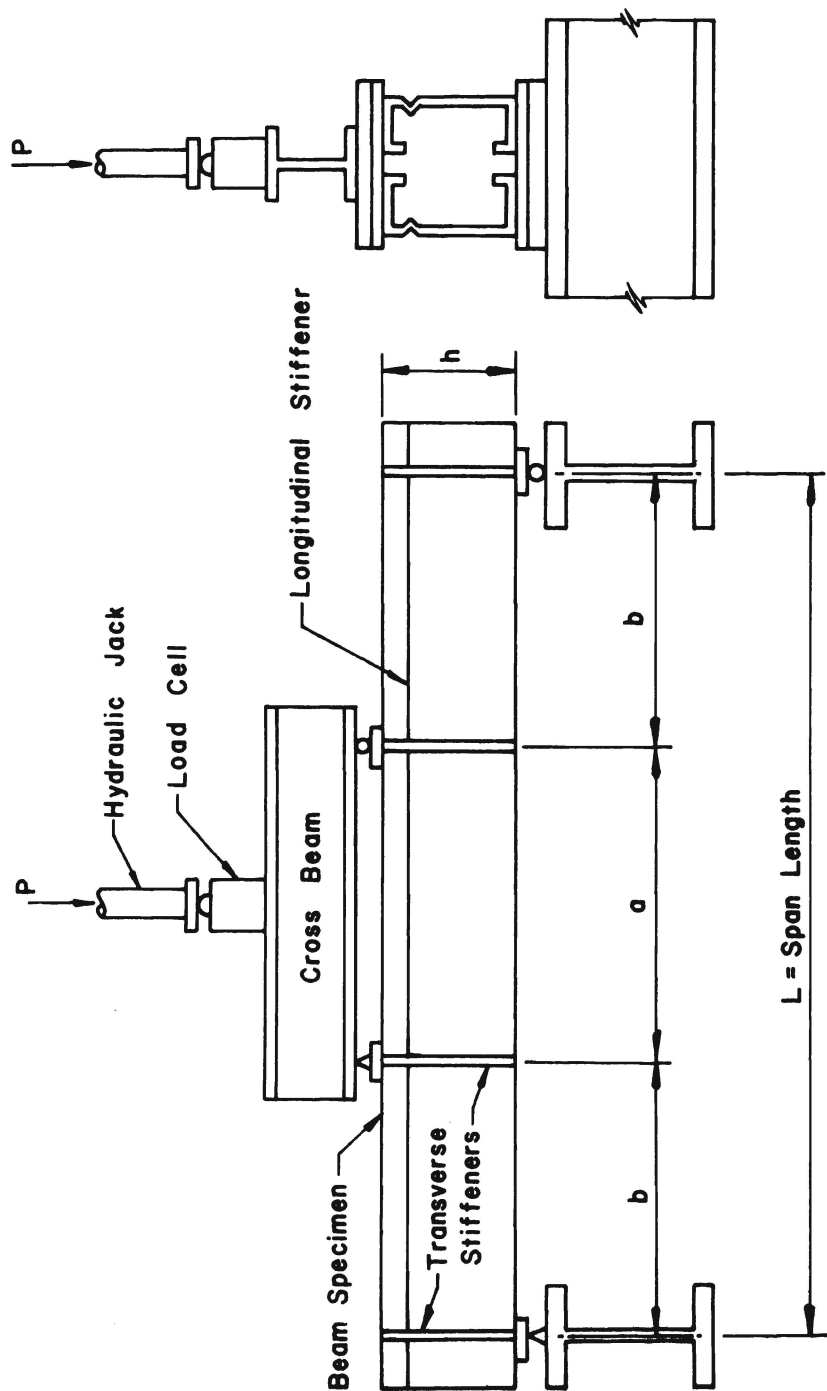


Fig. 5 Test Setup

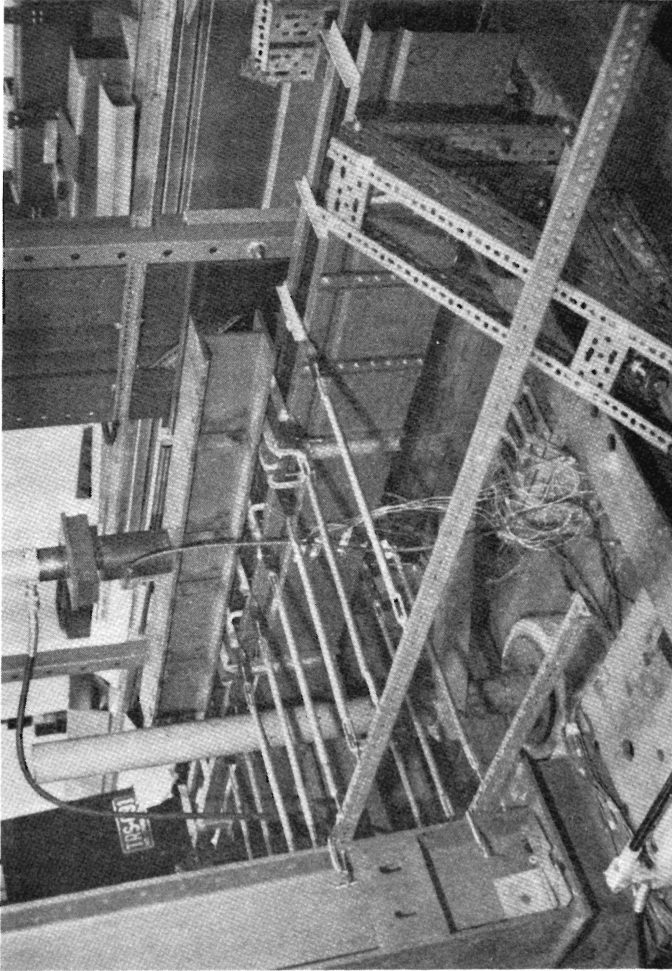


Fig. 6 Test Setup (Photo)

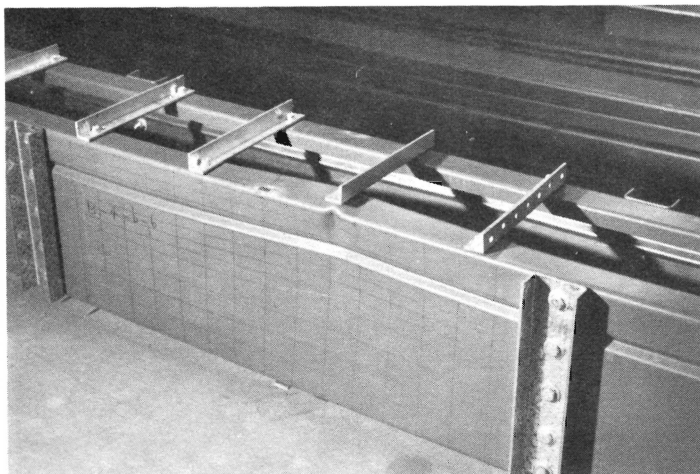


Fig. 7 Typical Failure Pattern for Channel Specimens (Photo)



Fig. 8 Typical Failure Pattern for Hat Specimens (Photo)